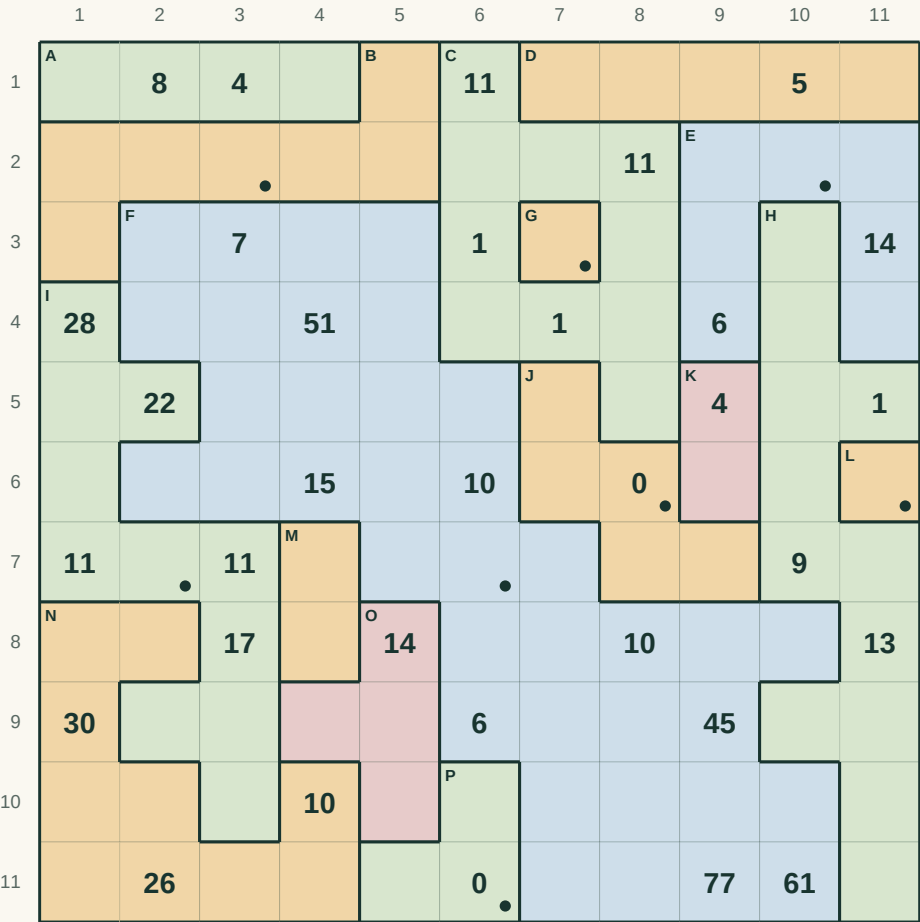


Three steps to the capitol

Why two three-step routes to one capitol cost 21 and 11.

From the 11 at (1,6), two three-step routes reach the capitol at (3,7), costing 21 and 11. The cheaper route leaves its 10-square state for a 5-square one and comes back. Each step costs the smaller of its two endpoint areas, so a small neighbor cuts the toll.



THE RESULT

408,951

- 16 connected, symmetric states.
- 8 capitol.
- All 33 clues reproduced.

Black dots mark capitol. Letters label states and match the audit table. Numbers are the original clues.

The map came from an AI-assisted CP-SAT search. A separate program checked its symmetries and shortest paths.

A symmetric state can still lack a capitol. Every nonidentity symmetry must fix exactly one square. A straight tromino has a half-turn center, but its reflection along the long axis fixes all three squares, so it gets none.

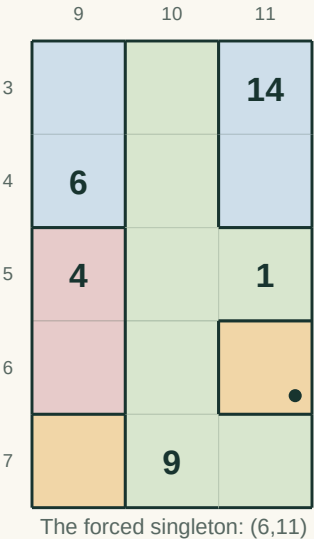
The puzzle: janestreet.com/puzzles/it-s-a-metric-too-index/

The 1, and the cheaper 11

A singleton forced by the clue at (5,11)

Every edge costs at least 1, so a clue of 1 reaches a capitol in one step, through a one-square state. The clue cell has positive distance, so the singleton must be its neighbor, and a singleton is its own capitol.

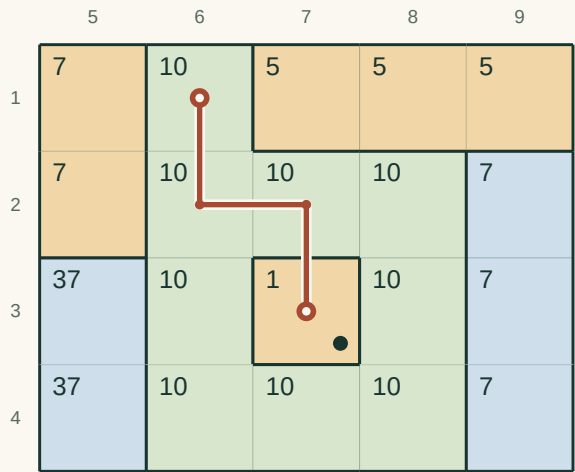
Apply this at (5,11). A singleton at (4,11) would put the 14 at (3,11) one step from a capitol. A singleton at (5,10) would do the same to the 4 at (5,9). No cell lies to the right. That leaves (6,11).



Two routes to the same capitol

From (1,6), both routes reach (3,7) in three steps. Going down, right, down costs 21. Going right, down, down visits the five-square bar at (1,7), comes back into the original state at (2,7), and costs 11.

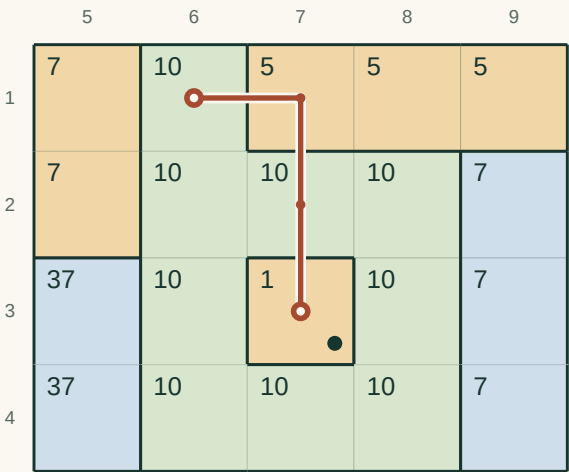
STAY INSIDE / COST 21



$10 + 10 + 1 = 21$

Cells show state areas; dots mark capitols.

CROSS THE BORDER / COST 11



$5 + 5 + 1 = 11$

Cells show state areas; dots mark capitols.

The saving has an exact form. Since $\min(a,b) = (a + b - |a - b|) / 2$, every border crossing charges the average of the two areas minus half their difference. Here both crossings cost 5 rather than 10.

Ruling out a cheaper path

Let $a(v)$ be the area of the state containing cell v . Join orthogonally adjacent cells by an undirected edge of weight $w(u,v) = \min(a(u), a(v))$. Each clue is the distance to the nearest capitol.

THREE CONDITIONS ON THE DISTANCE FIELD

A candidate field d gives the true distances if three things hold: (1) d is nonnegative and zero exactly at the capitols; (2) every edge satisfies $|d(u) - d(v)| \leq w(u,v)$; (3) every non-capitol v has a neighbor u with $d(v) = w(v,u) + d(u)$.

Condition (2) telescopes along any route from v to a capitol, so no route costs less than $d(v)$. Condition (3) builds a route by stepping to the tight neighbor each time. Each step lowers d by a positive amount, so the walk ends at a capitol with cost exactly $d(v)$.

	1	2	3	4	5	6	7	8	9	10	11
1	12	8	4	8	12	11	6	11	10	5	10
2	14	7	0	7	14	11	1	11	7	0	7
3	21	14	7	14	11	1	0	1	8	7	14
4	28	33	44	51	21	11	1	11	6	13	8
5	33	22	33	52	48	11	6	5	4	6	1
6	22	11	22	15	47	10	5	0	2	1	0
7	11	0	11	13	15	0	10	5	4	9	1
8	20	10	17	15	14	18	47	10	9	21	13
9	30	20	18	14	10	6	43	47	45	33	25
10	40	30	20	10	6	3	6	43	80	45	37
11	36	26	16	6	3	0	3	40	77	61	49

THE FULL DISTANCE FIELD

The 33 given clues are a sample of these 121 values. The verifier checks the bound on all 220 grid edges and finds a tight downhill step at every positive cell.

None of this relies on the solver. The verifier rebuilds the graph from the partition alone and runs multi-source Dijkstra from the capitols.

The 77 at (11,9), for instance, walks left along the bottom row: 37 + 37 + 3 to the capitol at (11,6).

The certificate covers this partition only. Nothing here shows that it is the only one.

The row calculation

Replace each cell by the area of its state, add up each row, then square the eleven row totals. Squaring individual cells instead would give a different number.

	1	2	3	4	5	6	7	8	9	10	11	ROW	SUM	SQUARED
1	4	4	4	4	7	10	5	5	5	5	5	1	58	3,364
2	7	7	7	7	7	10	10	10	7	7	7	2	86	7,396
3	7	37	37	37	37	10	1	10	7	12	7	3	202	40,804
4	11	37	37	37	37	10	10	10	7	12	7	4	215	46,225
5	11	11	37	37	37	37	5	10	2	12	12	5	211	44,521
6	11	37	37	37	37	37	5	5	2	12	1	6	221	48,841
7	11	11	11	2	37	37	37	5	5	12	12	7	180	32,400
8	10	10	11	2	4	37	37	37	37	37	12	8	234	54,756
9	10	11	11	4	4	37	37	37	37	12	12	9	212	44,944
10	10	10	11	10	4	3	37	37	37	37	12	10	208	43,264
11	10	10	10	10	3	3	37	37	37	37	12	11	206	42,436
TOTAL													408,951	

STATE REGISTER

- A

 area 4 / no capitol
- B

 area 7 / capitol (2,3)
- C

 area 10 / no capitol
- D

 area 5 / no capitol
- E

 area 7 / capitol (2,10)
- F

 area 37 / capitol (7,6)
- G

 area 1 / capitol (3,7)
- H

 area 12 / no capitol
- I

 area 11 / capitol (7,2)
- J

 area 5 / capitol (6,8)
- K

 area 2 / no capitol
- L

 area 1 / capitol (6,11)
- M

 area 2 / no capitol
- N

 area 10 / no capitol
- O

 area 4 / no capitol
- P

 area 3 / capitol (11,6)

Checksum: the row totals add to 2,033, which also equals the sum of the squared state areas.

Run `python3 verify.py solution.json` to check the states, capitols, all 33 clues, and the arithmetic. Coordinates are row, column, starting at 1. I used AI assistance for the search, code, and writing. The verifier and full writeup are at jsoc.t.rishabhsai.com.